CONVEX HULL - PARALLEL AND DISTRIBUTED ALGORITHMS

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Overview

1. Ultimate Planar Convex Hull Algorithm

2. Quick Hull Algorithm

Ultimate Planar Convex Hull

- Recursive algorithm employing the divide and conquer approach
- Computes the upper convex hull and lower convex hull
- Divides the space into two halves and finds the edge of upper (lower) convex hull cutting across the half

Ultimate Planar Convex Hull - Sequential and Parallel

- Sequential
  - n - number of points, h - number of edges in the convex hull
  - Recurrence is $f(n,h) = cn + \max_{h_l + h_r = h} \left(f(\frac{n}{2}, h_l) + f(\frac{n}{2}, h_r) \right)$
  - Upper Hull - $\mathcal{O}(nlogh)$ - Lower Hull
  - Overall Work (Worst Case) - $\mathcal{O}(nlogh)$
  - Scales with n and h
- Parallel
  - Recurrence is $f(n,h) = clog^3 n + \max_{h_l + h_r = h} \left(f(\frac{n}{2}, h_l), f(\frac{n}{2}, h_r) \right)$
  - Overall Depth (Worst Case) - $\mathcal{O}(log^4 n)$

Ultimate Planar Convex Hull - Distributed

- Not amenable to distributed scenario
- Divide and conquer paradigm - generally not good for distributed systems
- Involves call to a recursive function inside a recursive function

Quick Hull

- Approach similar to QuickSort
- Recursive algorithm - divides the space into subsets of points
- Removes points which doesn't belong to the convex hull

Quick Hull - Sequential and Parallel

- Sequential
  - Each call performs $\mathcal{O}(n)$ work and h such calls
  - Overall Work (Worst Case) - $\mathcal{O}(nh)$
  - Scales with n and h
- Parallel
  - Each call performs $\mathcal{O}(logn)$ work and h such calls
  - Not amenable to parallelization - in the h dimension
  - Overall Work (Worst Case) - $\mathcal{O}(hlogn)$
  - Scales with n and h

Quick Hull - Distributed

- Communication Pattern
  - All to One and One to All - All Reduce
- Communication Cost
  - m - number of machines, $\frac{n}{m}$ - data per machine
  - In each call, $\mathcal{O}(m)$ communications
  - h rounds, so $\mathcal{O}(mh)$ total communications
  - Scales only with h
- Work - $\mathcal{O}(\frac{n}{m}h)$
- Depth - $\mathcal{O}(log(\frac{n}{m})h)$

The End